Triamudomsuksa Pattanakarn School

Title

The Number Base Game

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Preface

This project, ‘The Number Base Game’, stemmed from our deep interest in the well-known birthday game: a game focused on the accurate guessing of an opponent’s date of birth. Our adoration of the birthday game, combined with us being math enthusiasts, led us to create an instructional media in an interactional format whilst maintaining the same focus of guessing number.

Fundamentally, our project is about the number base theory and games. The goal is to create an astonishing, logical, captivating and tricky math game. In addition, our target is to allow high school students to easily understand mathematics whilst also sparking further interest in the subject. That being said, our hope is for this game to become an alternative way for many students to understand mathematics. Subsequently, we intend for this game to have a useful outcome for everyone involved.

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Project

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Abstract

The birthday guessing game is an amazing game which uses number base 2 in making. According to this game, we try to think and look at it in a different view. We want to increase the complexity by changing into higher base. Normally, we play the game by using pieces of paper. In this work we try to make it look more interesting by putting number on geometric shape. As we were working on this, we found that there is some relation between number base and shape which is not going to make this game boring. The result is we found the relation between number base and shape. We could make use of this by using it as instructional media.

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Heading

Chapter 1 : Introduction

1.1. Background, Concept and Importance

The concept for the number base game came from our love for mathematics and our interest in games. Additionally, our observation during math class led us to believe that the number base topic is not very easy for everyone. That being said, we thought to make an instructional media to assist with the learning process, whilst sharing our passion for the topic. Overall, this game aims to reduce the number of students struggling to understand the topic of number base.

Our team discussed and developed an interesting game that used the number base as a fun trick. With the above mentioned reasons in mind, we attempted to look at the birthday game from a different point of view. We started by using higher numbers and changing its shape, to form some difficulty and attraction.

Normally, the game uses numbers from 1 to 31, which are usually written on pieces of paper. Though logical, people tend to get bored easily when they see lots of numbers within a page of paper. With that in mind, we decided to change the presentation and display the game in the form of a cube. Unfortunately, a cube has 6 sides which would make the numbers become too high. This is now a hurdle where more thought on our ideas begin.

1.2. Objectives

1.2.1. To study more about the tricks and usages of the number base.
1.2.2. To design an effective game utilizing number base knowledge.
1.2.3. To make an interesting instructional media focused on the number base.

1.3. Hypothesis

If we change the geometric shape, we could increase the number base range into a higher base. This number range can still be limited which can keep the players engaged while preventing the players from getting bored.

1.4. Variables

1.4.1. Independent variables : Geometric shapes, Number base
1.4.2. Dependent variables : Complexity of game
1.4.3. Control variables : Number range

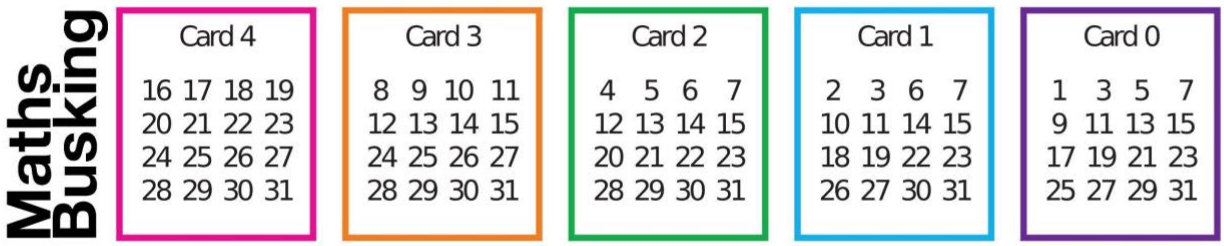
1.5. Expected Benefits

1.5.1. We will have alternative to learn the number base.
1.5.2. Changing into higher numbers and changing its shape will form some difficulty and attraction of this game.

Chapter 2 : Related Literature

2.1. Maths Busking- Mind Reading

The volunteer calls out numbers of the cards containing their birthday. If the birthday falls on the 22nd the person will chose cards 4, 2 and 1. You add up the first number printed on each of those cards. In this example the numbers are 16 (card 4), 4 (card 2) and 2 (card 1) yielding 16 + 4 + 2 = 22, the birthday (Maths Busking, 2014).



Note that the first numbers on the cards 0 to 4 are in the order of: 1, 2, 4, 8 and 16. Also notice the doubling pattern; such numbers are also called powers of two. This technique works for any number between 1 and 31. To perform this trick, one must download the card and print it double-sided.

Tip- if you want to remember the first number on each card, note the following:

2^2= 2x2 = 4 (card 2)

2^3= 2x2x2 = 8 (card 3)

2^4= 2x2x2x2 = 16 (card 4).

Also: 2^1 = 2, and 2^0= 1 (for cards 1 and 0).

When you select cards containing your birthday, you are in fact writing the date in binary. Surprised?

The numbers we use in our day-to-day life are written using ten digits from 0 to 9. This is called 'decimals' where each place in the number is worth units of tens, hundreds, thousands and so on. Consequently, these numbers are powers of 10, 1, 10, 102, 103.... For example, 546 in decimals represent 6 units, 4 tens and 5 hundreds.

In binary, we use 0s and 1s only, and each place in a number is worth units of twos, fours, eights and so on: doubling every time. For example, the number 1011 in binary is (from right to left):

1 unit plus 1 two plus 0 fours plus 1 eight, so 1+2+8 =11.

The fundamental part is that any number between 1 and 31 has a unique sequence of 0s and 1s in binary. Since we are using five places to write a number, all possible strings of 0s and 1s from 00001 to 11111 will give you 31 different possibilities.

Chapter 3 : Procedure

According to the problem, we will use the number 1 to 31 to make the game as days in a month.

Next, we change number base and see how many block we need to put the number in. We may look for general form for each one to generalize it. We start with base 2.

Let number of block be m

Base 2:

1

1	2

1 3 5 7	2 3 6 7	4 5 6 7

1 3 5 7 9 11 13 15	2 3 6 7 10 11 14 15	4 5 6 7 12 13 14 15	8 9 10 11 12 13 14 15

1 3 5 7 9 11 13 15 17 19 21 23 25 27 26 31	2 3 6 7 10 11 14 15 18 19 22 23 26 27 30 31	4 5 6 7 12 13 14 15 20 21 22 23 28 29 30 31	8 9 10 11 12 13 14 15 24 25 26 27 28 29 30 31	16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

Base 2	Block	Max
	1	1
	2	3
	3	7
	4	15
	5	31

The general form $2^m - 1$.

Base 3 :

1

1	2

From here, we need to separate 3 from first row because base 3 has 0, 1, and 2 as its base digit. We cannot choose it at the same time. For example, if we choose 3, which is also 2+1, this will create another answer that will not make us guess the number correctly.

1 4	2 5
3 4 5	

1 4 7	2 5 8
3 4 5	6 7 8

1 4 7 10 13 17	2 5 8 11 14
3 4 5 12 13 14	6 7 8 15 16 17
9 10 11 12 13 14 15 16 17	

Base 3	Block	Max	
	1	1	which is $2 \times 3^{\frac{1-1}{2}}$ -1
	2	2	which is $3^{\frac{2}{2}}$ -1
	3	5	which is $2 \times 3^{\frac{2-1}{2}}$ -1
	4	8	which is $3^{\frac{4}{2}}$ -1

We can divide in to two parts, which are odd and even.

For odd number of blocks, the general form is $2 \times 3^{\frac{m-1}{2}}$ -1.
For even number of blocks, the general form is $3^{\frac{m}{2}}$ -1 .

Base 4 :

Base 4 we can use modulo to help find the max number.

From base 2 and base 3, we can imply that the maximum number of that block is first number of next page subtracted by 1.

$m \equiv 0 \ (mod\ 3)$

Max = $4^{\frac{m}{3}}$-1

$m \equiv 1 \ (mod\ 3)$

Max = $2 \times 4^{\frac{m-1}{3}}$-1

$m \equiv 2 \ (mod\ 3)$

Max = $3 \times 4^{\frac{m-2}{3}}$-1

Mod3 $m \equiv 1$	Mod3 $m \equiv 2$	Mod 3 $m \equiv 0$
1	2	3
4	8	12
16	32	48

We try to write general form for base 4 as

Max = $(r+1) \times 4^{\frac{m-r}{3}}$-1 , if $m \equiv r \ (mod\ 3)$

Chapter 4 : Results

As we continue doing further from procedure above, we found the general form from base 4 as

$$\text{Max} = (\text{r} + 1) \times 4^{\frac{m-r}{3}} - 1 \quad \text{if } m \equiv r \ (mod\ 3)$$

From the general forms for the base 2, 3 and 4, we find that the general form for base n in term of

$$\text{Max} = (\text{r} + 1)n^{\frac{m-r}{n-1}} - 1 \quad \text{, if } m \equiv r \ (mod\ n - 1)$$

where r is remainder,

m is number of block,

n is number base.

The next step is to find the number of face of geometric shapes that suits each base number. As we can see from the result of base 2, if we put 5 blocks in to it, there will be too many numbers to play. It makes game boring.

1 3 5 7 9 11 13 15 17 19 21 23 25 27 26 31	2 3 6 7 10 11 14 15 18 19 22 23 26 27 30 31	4 5 6 7 12 13 14 15 20 21 22 23 28 29 30 31	8 9 10 11 12 13 14 15 24 25 26 27 28 29 30 31	16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

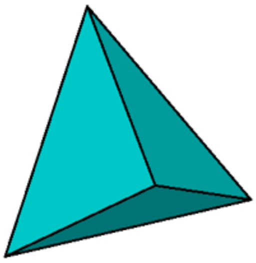
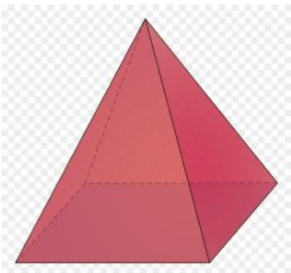
We try on higher bases and higher numbers of blocks and limit the number as we mentioned. We get the relation as follow:

Block \ Base	2	3	4	5	6
4	✓	✗	✗	✗	✗
5	✓	✓	✗	✗	✗
6	✗	✓	✓	✗	✗
7	✗	✗	✓	✓	✗
8	✗	✗	✗	✓	✓

✓ **: Suitable**
✗ **: Unsuitable**

Now, we decide which geometric shape is suitable for each number base.

In reality, we found out that not every geometric shape has the same number of faces as the number of blocks. Normally, we only found geometric shapes that have less than 7 sides. For example, a triangular pyramid has 4 faces, a square pyramid has 5 faces and a cube has 6 faces.

Geometric Shape	**Base**
	2
	2, 3
	3, 4

Chapter 5 : Conclusion

After we increase the difficulty by faces and base number, we find the general form of base 2, 3 and 4 as follows.

Base 2	$2^m - 1$
Base 3 Odd ($m \; mod \; 1 = 1$)	$2 \times 3^{\frac{m-1}{2}}$ -1
Base 3 Even ($m \; mod \; 2 = 0$)	$3^{\frac{m}{2}}$ -1
Base 4 ($m \; mod \; 3 = r$)	$(r + 1) \times 4^{\frac{m}{3}}$-1

From this general form, we get the general form of the maximum number of n base as

$$\text{Max} = (\text{r} + 1)n^{\frac{m-r}{n-1}} - 1 \quad , \quad \text{if} \quad m \equiv r \; (mod \; n - 1)$$

where r is remainder,

m is number of page,

n in number base.

As we get the general form for base n, we can now find the number of faces that is suitable for each number base when the maximum or minimum of the number is in range of 15 to 25.

Base / Block	2	3	4	5	6
4	✓	✗	✗	✗	✗
5	✓	✓	✗	✗	✗
6	✗	✓	✓	✗	✗
7	✗	✗	✓	✓	✗
8	✗	✗	✗	✓	✓

✓ **: Suitable**
✗ **: Unsuitable**

According to the formula where r is the remainder, m is the number of blocks, and n is the number base, we decided to put base 2 numbers in a triangular pyramid and a square pyramid, which has 4 faces and 5 faces respectively. For base 3 numbers, we put them in a square pyramid and a cube, which contain 6 faces. We also put base 4 numbers in a cube.

In conclusion, the result from increasing the number base and the number of the blocks, we found the relation between number base and number of block. Then, we found the suitable geometric shapes which follow the condition. Finally, we increase the complexity of this game and create an effective instructional media.

Bibliography

Reference

- *Maths Busking*. (2014, March 23) from http://www.mathsbusking.com/shows-mind-reading.html